Rogers, Alice
Supermanifolds. Theory and applications. (English) Zbl 1135.58004
Singapore: World Scientific (ISBN 978-981-02-1228-5/hbk). xii, 251 p. (2007).

Supermathematics is flourishing. The earliest step in supermathematics was *É. Cartan*'s [La théorie des groupes finis et continus et la géométrie différentielle traitees par la méthode du repere mobile. Paris: Gauthier-Villars (1937; Zbl 0018.29804, JFM 63.1227.02)] recognition that a Clifford algebra could be represented on a Grassmann algebra if one includes a notion of differentiation with respect to a generator as well as multiplication, which was to reappear decades later in connection with fermion anticommutation relations. A supersymmetric theory enjoys invariance with a symmetry exchanging bosonic and fermionic degrees of freedom, so that fermions and bosons should be dealt with on an equal footing. There exist in the literature a confusing number of different formulations of the concept of a supermanifold. Apart from differences of detail, there are two completely different ways of looking at supermanifolds. In the concrete approach, it is a manifold modelled on some flat superspace so that it has local coordinates some of which take values in the even and some in the odd part of a Grassmann algebra. In the algebro-geometric approach, it is the sheaf of functions on a manifold which is extended.

This book presents both approaches, but it is decisively inclined towards the concrete approach. The concrete approach is presented in Chapter 5, where a useful working definition of the G^∞ DeWitt supermanifolds is given and related to other definitions. Chapter 6 lays the foundations of differential geometry on supermanifolds by introducing the concept of a supersmooth or G^∞ function on a supermanifold, which is followed by the notions of a G^∞ mapping between supermanifolds, a tangent vector, a vector field, and so on. The algebro-geometric approach is taken up in Chapter 7, where the treatment is very brief, because *M. Batchelor*'s result [Trans. Am. Math. Soc. 258, 257–270 (1980; Zbl 0426.58003)] on the equivalence of both approaches is discussed in Chapter 8. One of the principal motivations for studying supermanifolds is the desire to find the appropriate global object corresponding to a super Lie algebra. Therefore it is natural that the author should devote Chapter 9 to super Lie groups, which are supermanifolds and groups at the same time, while neither the formal groups of *F. A. Berezin* and *G. I. Kac* [Mat. Sb., N. Ser. 82(124), 343–359 (1970; Zbl 0244.22014)] nor graded Lie groups of *B. Kostant* [Differ. Geom. Meth. Math. Phys., Proc. Symp. Bonn 1975, Lect. Notes Math. 570, 177–306 (1977; Zbl 0358.53024)] are really groups.

The book, consisting of 17 chapters, discusses many geometric ideas on supermanifolds and their applications to physics, and the reviewer believes that it will be the standard reference in the theory of supermanifolds for decades.

Reviewer: Hirokazu Nishimura (Tsukuba)

MSC:

58A50	Supermanifolds, etc. (global analysis)
58-01	Textbooks (global analysis)
32C11	Complex supergeometry
58C50	Analysis on supermanifolds or graded manifolds